# The scaling approach

**Scaling** is an approach were we find multiples of a ratio.

For example, if we know that two people's ages are in the ratio 2:3, think about how old they could be (assuming integer ages):

2 : 3

4 : 6

6 : 9

8 : 12

10 : 15

12 : 18

14 : 21

and so on (ie their ages could be any multiple of 2:3).

There are between 25 and 35 students in a class. The ratio of boys to girls is 4:7. How many students are there in the class?

Note: Except for scaling we can't solve this problem algebraically……

Scaling up from 4 : 7, we could have:

8 : 14

12 : 21

16 : 28

etc

**We're told that the total is between 25 and 35.**

# Scaling and Algebraic methods to solve Ratio problems

1. There are 35 students in the class and the ratio of boys to girls is 3:4. Calculate the number of girls.

| Scaling Method | Algebraic Method |
|---|---|
| Ratio 3:4<br>Equivalent ratios are<br>6:8<br>9:12<br>12:16<br>15:20 and so on…….<br>Since total no of students are given as<br>35….we stop here as 15 + 20 = 35<br><br>Hence No of girls are 20… | Let No of boys be 3x<br>No of girls be 4x<br><br>Total No of st's = 35<br><br>3x + 4x = 35<br>7x = 35<br>x = 5<br><br>Hence No of girls = 4 x 5 = 20 |

**Yash and Rohith share some sweets in the ratio 7 : 3. Yash gives 3 sweets to Rohith and the ratio now becomes 5 : 3 , How many sweets did each have initially.**

| Scaling method | Algebraic Method |
| --- | --- |
| Original Ratio 7:3<br>On scaling<br>14:6 Y gives 3 to R…so 11: 9<br><br>21:9…… so 18 : 12 = 3:2<br><br>28:12……so 25 : 15 = 5:3<br><br>Hence originally Yash has 28 sweets<br>Rohith has 15 sweets | *Initially, Alice had 7x sweets and Olivia had 3x sweets*<br><br>*Alice gives 3 sweets to Olivia*<br><br>*So Alice now has* $7x - 3$ *sweets and Olivia has* $3x + 3$ *sweets*<br><br>*The ratio 7x - 3 : 3x + 3 is 5:3*<br><br>$\frac{7x-3}{3x+3} = \frac{5}{3} \quad \therefore \quad 3(7x-3) = 5(3x+3)$<br><br>*Solve to get* $x = 4$<br><br>$\therefore$ *initially Alice had 28 sweets and Olivia had 12 sweets* |

**In a bag, the ratio of red to blue counters is 3 : 4. If 3 red counters are removed the ratio of red to blue counters becomes 3 : 5. How many blue counters are there in the bag?**

| Scaling Method | Algebraic Method |
|---|---|
| red : blue = 3: 4<br>3 red are removed<br>Using scaling 6 : 8 ---------- 3: 8<br>9: 12 ..... 6 : 12<br>12: 16. ..... 9 : 16<br>15 : 20 ..... 12 : 20....which is 3:5<br>Hence no of original counters are 35 | Let red and blue counters be 3x and 4x<br>$\frac{3x-3}{4x} = \frac{3}{5}$<br>$5(3x-3) = 3(4x)$<br>$\therefore 15x - 15 = 12x$<br>$\therefore x = 5$<br>Hence red counters = 15, Blue = 20<br>Total counters 35 |

Punch is made by mixing Orange juice and cranberry juice in the ratio 7 : 2. Arya has 30 litres of Orange juice and 8 litres of Cranberry juice …What is the maximum amount of punch that Arya can make ?

- Using Scaling approach , multiplying the punch ratio by 4 gives us 28:8

- Hence the most punch we can make is 36 litres of punch

# “At present, Bob is twice as old as Alice. In 20 years time, the ratio of Alice to Bob’s age will be 9: 13. How old is Alice now?”

B = 2A. …..Hence  A : B = 1 :2

Here it’s easier to start with 9 : 13, because starting with 1:2 takes ages.

In 20 years their ages could be:

9 Alice  13 Bob
18 Alice  26 Bob
27 Alice  39 Bob
36 Alice  52 Bob
45 Alice  65 Bob
54 Alice  78 Bob
etc

Then subtract 20 from both parts to find a ratio equivalent to 1 : 2.

-11 Alice  -7 Bob
-2 Alice  6 Bob
7 Alice  19 Bob
16 Alice  32 Bob
25 Alice  45 Bob
34 Alice  58 Bob

# This question could have been solved using scaling instead of algebra and equivalent fractions.

The ratio of the number of boys to girls at a party is 3 : 4

Six boys leave the party.

The ratio of the number of boys to girls at the party is now 5 : 8

Work out the number of girls at the party.

It would have been quicker to start with multiples of 5:8 and add 6 to the number of boys

Initially we had a ratio of boys to girls of 3 : 4. So this might have been 3 boys and 4 girls, or 6 boys and 8 girls etc. We could list some options:

3 boys 4 girls
6 boys 8 girls
9 boys 12 girls
12 boys 16 girls
15 boys 20 girls
18 boys 24 girls
21 boys 28 girls
24 boys 32 girls
27 boys 36 girls
30 boys 40 girls
33 boys 44 girls
36 boys 48 girls
etc

If six boys leave then we end up with the following numbers:

-3 boys 4 girls
0 boys 8 girls
3 boys 12 girls
6 boys 16 girls
9 boys 20 girls
12 boys 24 girls
15 boys 28 girls
18 boys 32 girls
21 boys 36 girls
24 boys 40 girls
37 boys 44 girls
30 boys 48 girls

From this we can identify the new ratio of 5 : 8.

# Exercise

ratio problems (i)

(1) the ratio of shares owned by Asterix and Cleopatra is in the ratio 3 : 5

if Asterix gives Cleopatra 2 of his shares then the ratio will be 1 : 3

how many shares do they each have initially?

(2) Jan and Kim own numbers of marbles that are in the ratio 5 : 6

Jan gains 2 more marbles and the ratio is now 7 : 8

how many marbles do each own initially?

(3) the ratio of Ann's age to Bob's age is 3 : 4

in 7 years time this ratio will be 4 : 5

(i) what are their ages now?
(ii) after how many years (from now) will the ratio be 5 : 6?

(4) the ratio of cockles to winkles in a bag of sea shells is 1 : 2

when 6 winkles are accidentally spilt from the bag the ratio is 3 : 5

how many of each shell were there initially?

(5) the ratio of geese to ducks on a lake is 4 : 9

when 10 extra geese land on the lake (and there are no duck changes) the ratio is 2 : 3

how many of each were there initially?

(6) the ratio of the width to depth (when they are measured in metres) of a dam is 5 : 7

when 10 extra metres is added to the width of the dam (and the depth remains the same) the ratio is 5 : 6

how wide and deep was the dam initially?

Same Exercise, Different Method!

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Questions by Don Steward