Teaching & Learning Plans

Plan 8: Introduction to Trigonometry

Junior Certificate Syllabus

Project Maths Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. Student Learning Tasks – Teacher Input: This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. Student Activities – Possible and Expected Responses: Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. Teacher's Support and Actions: Gives details of teacher actions designed to support and scaffold student learning.

iv. Checking Understanding: Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 8: Introduction to Trigonometry

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Aims

- To introduce the concept of trigonometry
- To understand the concept of sin, cos and tan
- To apply trigonometry to solve problems

Prior Knowledge

Students should have knowledge of the concept of ratio and should know that division is not commutative. Students should be able to use a calculator to convert fractions, correct to three decimal places, measure the lengths of sides of a triangle to the nearest millimetre and draw to scale. Students should have studied Pythagoras' Theorem and know the meaning of the term "hypotenuse". Students may have studied similar triangles in geometry – the fact that corresponding sides are proportional makes trigonometry possible. Students will have understood how to calculate mean. Students should know how to use a measuring tape and protractor.

Learning Outcomes

As a result of studying this topic, students will be able to

- correctly identify the hypotenuse in a right angled triangle, and the opposite and adjacent sides of a given angle
- find a pattern linking the ratio of sides of a triangle with the angles and hence understand the concepts of sine, cosine and tangent ratios of angles
- apply trigonometry to solve problems including those involving angles of elevation and depression

Relationship to Junior Certificate Syllabus

Sub-topics	Ordinary Level	Higher Level (also includes OL)
Trigonometry	Solution of right-angled triangle problems of a simple nature involving heights and distances, including the use of the theorem of Pythagoras. Cosine, sine and tangent of angles (integer values) between 0° and 90°.	Functions of 30°, 45° and 60° in surd form derived from suitable triangles. Solution of right-angled triangles. Decimal and DMS values of angles.

Resources Required

Calculators, measuring tapes, graph paper, cm rulers and a sunny day!

Introducing the Topic

Trigonometry is simply geometrical constructions where the ratio of the side lengths in triangles is used to determine angular measurement.

Trigon, meaning triangle and *metria*, meaning measurement

Mathematicians have used trigonometry for centuries to accurately determine distances without having to physically measure them (Clinometer Activity **Appendix A**). It can also be used to calculate angles that would be very difficult to measure. Trigonometry has uses in such areas as surveying, navigation, drawing and architecture.

Lesson Interaction			
Student Learning Tasks: Teacher Input	**Student Activities: Possible and Expected Responses**	**Teacher's Support and Actions**	**Checking Understanding**
Outside Classroom » We are going to find the angle of elevation of the sun. Fill in **Student Activity 1A.**	» Students measure one student's height and the length of his/her shadow and fill in **Student Activity 1A**. » Students measure the length of the shadow of some tall object e.g. flagpole or goalpost and fill in **Student Activity 1B**.	» Distribute **Student Activity 1** (sunny day necessary!) » Walk around observing students as they take measurements.	» Have all students understood the task so far? » Can students identify the angle of elevation?
Inside Classroom » Fill in **Student Activity 1C**. » From your scaled diagram work out the angle of elevation of the sun.	» Students fill in **Student Activity 1C** to find the angle of elevation of the sun and check their answer with other students in the class.	» Circulate checking that students can draw a scaled diagram.	
» Now using the angle of elevation (from **Student Activity 1C**) and the length of the shadow of an object whose height you cannot physically measure (from **Student Activity 1B**), again use a scaled diagram to determine the height of the object.	» Students fill in **Student Activity 1D** to find the measure of some tall object and check their answer with other students in the class.		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Alternatively, you can use ratios in similar triangles. Fill in **Student Activity 1E**. **Note:** Later on, having learned about the trigonometric ratios of sin, cos and tan studentscan use these trigonometric ratios to determine the angle of elevation of the sun and hence the height of for example a tall tree or a goalpost.	» Students complete **Student Activity 1E**.		» Have all students been able to complete **Student Activity 1E**?
» Why is the study of triangles important?– show students the PowerPoint images in **Appendix B.**			
» What is the hypotenuse of a right-angled triangle?	• The longest side • The side opposite the 90° angle		
» Collaborate in pairs as you fill in **Student Activity 2**. » Mark the hypotenuse on the triangles in **Student Activity 2** of right-angled triangles.	» Students can mark the hypotenuse of each right angled triangle.	» Distribute **Student Activity 2**. » Have the heading "Right angled triangles" on the board. (RHS of the board should have heading "New Words"). » Draw a large right angled triangle with the right angle marked. » Mark the hypotenuse on the board with an arrow to show that the side is opposite the right angle.	» Are all students able to identify the hypotenuse?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» How many other angles are in the triangle? » How many degrees do they add up to? » What are these angles called?	• Two • 90° • Complementary angles		
» Mark either one of the two complementary angles on the triangle with an arc. What is the name of one of the arms of that angle?	• The hypotenuse	» Label the hypotenuse on the triangle on the board.	
» The other side, which is beside the marked angle, is given the name "adjacent". Label it.	» Students mark one angle with an arc and label the adjacent side to this angle on all the triangles.	» Label the adjacent and write the word "Adjacent" in the new words list on RHS of board. » Tell students that the word adjacent means "beside". They are sitting beside or "adjacent to" another student.	
		» Point out on the board that regardless of which of the two angles adding up to 90° is chosen by the student, the hypotenuse is one of the arms of that angle. » Circulate checking that students can label the sides correctly; emphasising that the labelling of a side depends on the angle it refers to. Adjacent means adjacent to a particular angle.	» Can all students identify the side adjacent to the marked angle?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Label the third side of the triangle as the opposite side. » Describe the opposite side.	» Students label the side opposite the marked angle as the opposite. • If you were standing at the vertex of the marked angle it would be across from you or opposite you so it is called the opposite side.	» Check each group's work as they are labelling the sides of the triangle.	» Are all sides correctly labelled in each triangle?
» Repeat this process of marking angles and labelling sides for all the triangles on **Student Activity 1**.	» Students label 90° angle, the hypotenuse, one other angle then adjacent and then opposite sides.	» Circulate checking the work of each group.	» Are all sides correctly labelled in each triangle?
» If you were to work out the ratio of any two of the sides in a right angled triangle how would you do this? » Would the order matter? Explain your answer.	» Divide the length of one side by the length of another side using the same units. » Yes, because 3/2 is not the same as 2/3 for instance.	» Ask students to think about the answer first and then ask a student for an answer. » If a student cannot explain the answer, give two lengths and then ask the student if the order matters.	» Are students familiar with the concept of ratio and do they understand that division is not commutative?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» How many possible ratios could be worked out for a right-angled triangle? » When giving the ratios use the names for the sides.	• **6** $\frac{opp}{hyp}, \frac{hyp}{opp}$, $\frac{adj}{hyp}, \frac{hyp}{adj}$, $\frac{opp}{adj}, \frac{adj}{opp}$	» Write the ratios on the board as students call them out placing multiplicative inverses beside each other.	» Are the students able to take the 3 sides and combine them into 6 pairs?
» What is the relationship between the first pair? » If I knew the answer to the first ratio was ½, what would be the answer to the second one?	• One is the inverse of the other. If they were multiplied, they would give an answer of 1. • 2/1	» Circle $\frac{opp}{hyp}, \frac{adj}{hyp}$, and $\frac{opp}{adj}$ and tell students that you will concentrate on these 3 as the others are their multiplicative inverses.	» Do students know the concept of a multiplicative inverse?
» **Student Activities 3, 4, 5, 6, 7 and 8**: For each triangle of the five triangles mark the 90° angle and one other given angle (given on the sheet). Label the sides of the right angled triangles as hyp (hypotenuse), adj (adjacent), and opp (opposite). Measure each side correct to the nearest mm and calculate the ratios opp/hyp, adj/hyp, opp/adj. » One student is to measure, one to calculate ratios and one to double check changing roles on each triangle.	» Students work on **Student Activities 3, 4, 5, 6, 7 and 8**.	» Distribute **Student Activity 3, 4, 5, 6 ,7 and 8** to different groups of 3-4 students each i.e. one Student Activity per group. » Emphasise that students should measure accurately to the nearest mm.	» Are students measuring accurately and calculating ratios correctly i.e. opp/hyp and not hyp/opp?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Write down what you have observed from your answers.	• The ratios are unchanged for a particular angle regardless of the size of the triangle.	» If a group has difficulty seeing the pattern or verbalising it you will see this as you circulate and help by using leading questions.	» Are the students finding that the ratios are unchanged for a particular angle regardless of the size of the triangle?
» Of the 3 ratios which of them can never be bigger than 1? » Explain.	• $\frac{opp}{hyp}$, and $\frac{adj}{hyp}$ • Numerator will always be smaller than the denominator as the hypotenuse is the longest side.	» Ask the class when they have a few of the ratios calculated to answer this question and then ask one group to explain. » Circulate checking the progress of the different groups to see that sides are being labelled correctly and that students understand the task.	» Have they been able to answer this question based on their knowledge of right angled triangles?
» Is it possible for any of the ratios to be bigger than 1? » If so, which one or which ones and why?	• Yes • $\frac{opp}{adj}$ - The opposite is bigger than the adjacent when it is opposite the bigger of the 2 complimentary angles.	» If students cannot answer this question, give an example using numbers when fractions give answers greater than 1 or less than 1.	» Do students understand that the only one of the three ratios which can be bigger than 1 is $\frac{opp}{adj}$?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» We have been studying how the sides and angles of triangles are related to each other. This is called TRIGONOMETRY – Trigon meaning triangle and metria meaning measurement.	» Students write this heading and the ratios into their copies.	» Write the word TRIGONOMETRY on the board. » Tell the students that the ratios they have investigated have special names and write them on the board. » $\frac{opp}{hyp}$ = sin e of the angle $\frac{adj}{hyp}$ = cosine of the angle $\frac{opp}{adj}$ = tan of the angle	
» Go back to **Student Activities 3 - 8** and fill in the name of each ratio, for example for the sheet with angles of 30° fill in opp/hyp = sin 30° etc.	» Beside each ratio students fill in the appropriate name plus the angle it refers to.	» Tell students that sin is the short version of sine, cos is short for cosine and that tan is the shortened version of the word 'tangent'.	
» On the master table, **Student Activity 9** fill in the mean value you have calculated for sin, cos and tan of the angle from your own **Student Activity 3, 4, 5, 6, 7, or 8**. Tell the rest of the class the values you have calculated.	» Each group gives the answers for each ratio for the angle they have worked on.	» Hand out **Student Activity 9**. » Draw a master table on the board and fill in the answers as they are given telling students that they will be able to check those answers in the next step.	» Have all students understood the tasks so far, completed them for all the triangles, and filled out the master table?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What units are angles measured in?	• Degrees		
» There are other units for measuring angles such as radians, which you will meet later on so you must be sure your calculator is in degree mode if you are using degrees.		» Show students how to check if their calculator is in degree mode and, if not, how to put it into this mode.	
» Using the calculator check the values of sin, cos and tan of the angles, which you have calculated through measurement. Check the measurements of the rest of the class also.	» Students check the values in the master table with the values got using the calculator and fill in to "check" columns on the master table **Student Activity 9**.	» Emphasise that sin, cos and tan are functions of angles. » Circulate to see that the values of sin, cos and tan of angles calculated through measurements agree with those found on the calculator.	
» Using the answers on **Student Activity 9,** answer the questions on **Student Activity 10**.	» Students see patterns in the answers on the master table.	» Distribute **Student Activity 10**. » Circulate to ensure that students are able to see the patterns and are using correct terminology.	
» If you knew the ratios how would you find out the angle? Given that the sin of an angle is 0.5 how do you find the angle?	• Students may be familiar with the SHIFT or 2nd function button on the calculator and hence suggest using this button.	» Write on the board - Given a trig ratio, for example sin A = 0, the angle A = $\sin^{-1}(0.5)$. Emphasise that $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ represent angles where x is a ratio of sides in a right angled triangle.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Given $\sin^{-1}A=0.7$ find A $\cos^{-1}B=0.2$ find B $\tan^{-1}C=1.75$ find C	» Students use their calculators to evaluate these angles.		
Reflection » List what you have learned today.	• Trigonometry is about the study of the relationship between angles and ratio of sides in triangles. • The sides in a right angled triangle are labelled hypotenuse, and then adjacent and opposite depending on which of the two complementary angles is of interest. • The three ratios of sides in a right angled triangle are: $\frac{opp}{hyp}$ = sin e of the angle $\frac{adj}{hyp}$ = cosine of the angle $\frac{opp}{adj}$ = tan of the angle		
» Write down anything you found difficult. » Write down any questions you may have.		» Circulate and take note particularly of any questions students have and help them to answer them.	» Are students using the terminology with understanding?

Student Activity 1

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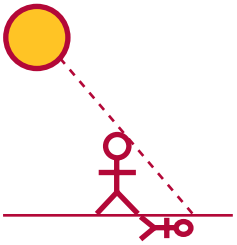
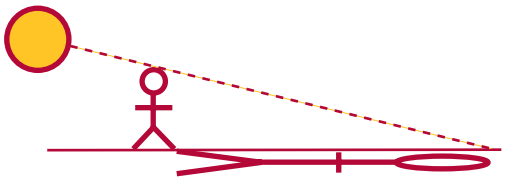
Me and my shadow

Safety warning: Never look directly at the sun

Name: _____ **Class:** _____

Date: _____ **Time:** _____

	
When the sun is high, your shadow is short.	When the sun is low, your shadow is long.

Student Activity 1A

- Show the angle of elevation of the sun on the above diagram. Call it A.
- Describe the angle of elevation of the sun in terms of the two arms of the angle._____
  _____
  _____
- Measure the height of one of the students in your group and the length of their shadow.
- Height of the student: _____cm. Length of the shadow _____cm.
- Draw a rough sketch of a right-angled triangle to model the situation and write in the measurements.

Student Activity 1B

Measure the length of the shadow of some tall object e.g. flagpole or goalpost. Length of the shadow of a tall object which you cannot physically measure e.g. goalpost

_____cm

Student Activity 1C

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Back in class – Measuring the angle of elevation of the sun

- Decide what scale to use.
- Draw an accurate diagram on graph paper.

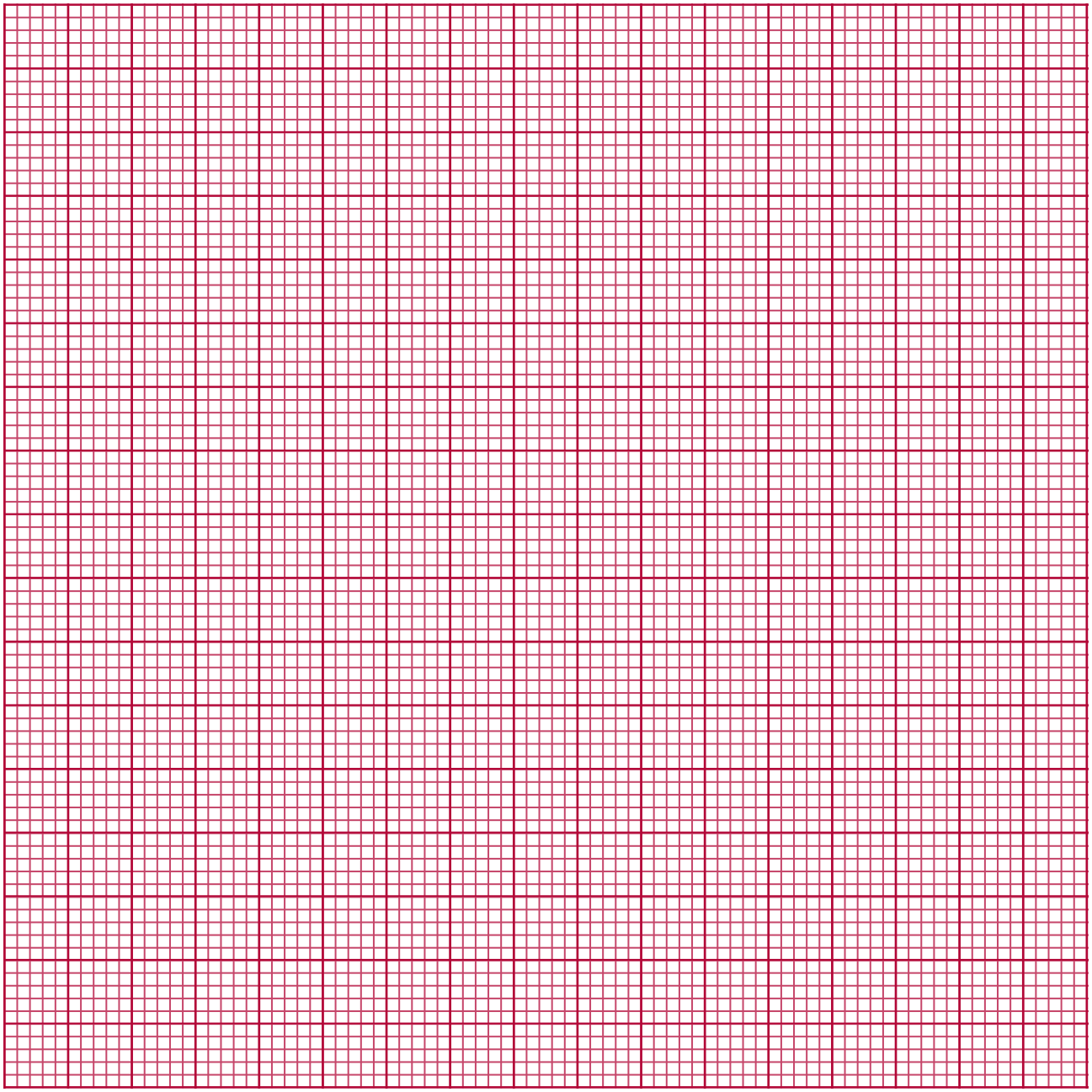

Diagram 1

- Measure the angle of elevation of the sun from Diagram 1 above using a protractor.
- Angle of elevation of the sun at _____ (time) on _____ (date) was _____.
- Check your answer with other students in the class.
- If you were to measure the angle of elevation of the sun at 10 a. m and another class measured the angle at 11 a.m. what would be the difference in the measurements?_____

Student Activity 1D

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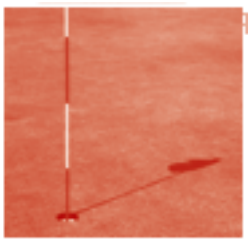
Knowing the angle of elevation of the sun, measure the height of a tall object using the length of its shadow as previously measured.



- Decide what scale to use. Scale: _____
- Draw an accurate diagram on graph paper using the length of the shadow, the angle of elevation of the sun and forming right-angled triangle (ASA).

Diagram 2

- Measure the height of the goalpost from Diagram 2 above and using the scale factor convert to its actual height.
- Check the answer with other students in the class.

Conclusion for part 2: The height of the goalpost is _____cm approximately.
Would you expect the same answer if you took the measurements at different times of the day?
Explain your answer._____

_____

_____

Student Activity 1E

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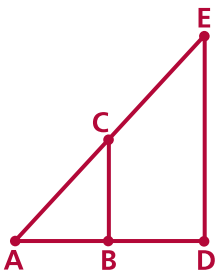
Using Similar Triangles

- Using graph paper draw the above 2 diagrams overlapping, with the angles of elevation of the sun superimposed as shown by example in the diagram on the right. Label the diagram on the graph paper as in the diagram on the right.



- What do you notice about the 2 vertical lines in the triangles? _____
  _____
- Measure the heights of the 2 vertical lines $|ED|$ and $|CB|$. $\frac{|ED|}{|CB|}$ = ____
- Measure the 2 horizontal lines $|AB|$ and $|AD|$. $\frac{|AD|}{|AB|}$ = _____
- What do you notice about the two ratios? _____
  _____
  _____
- Knowing $|AB|$ and $|CB|$ and the distance $|AD|$ how could you find $|ED|$ without knowing the angle of elevation $|\angle EAD|$ of the sun?

Student Activity 1E

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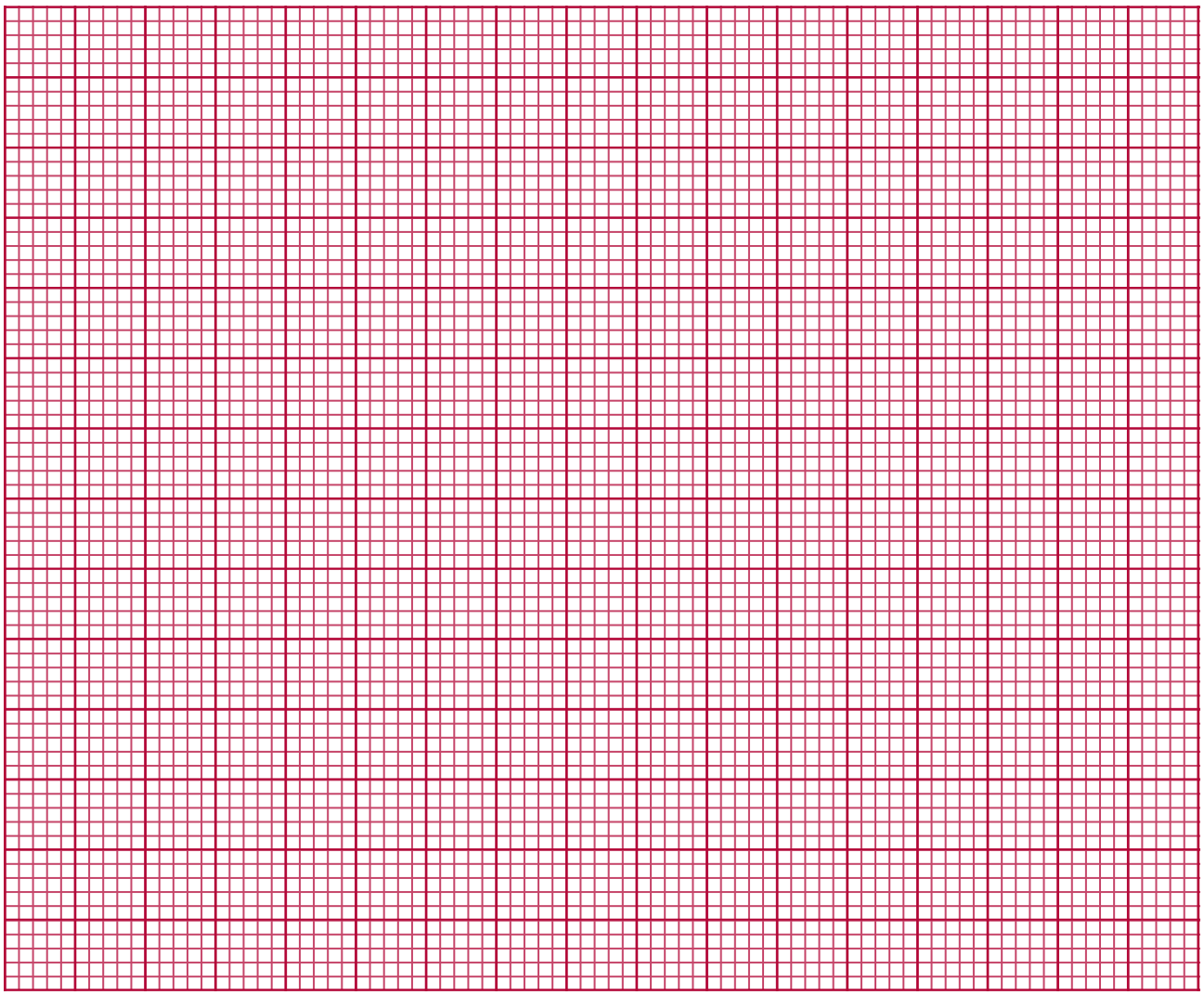
Ratios in Similar Triangles

- Draw 3 different right angled triangles with the arms of the 90 degree angle being vertical and horizontal line segments, using the same angle of elevation which you calculated for the sun. Call the triangles T1, T2, T3.



Measure the length of the vertical and horizontal line segments in these triangles.

Vertical T1	Horizontal T1	Vertical T2	Horizontal T2	Vertical T3	Horizontal T3
$\frac{\lvert V1\rvert}{\lvert V2\rvert} =$	$\frac{\lvert V1\rvert}{\lvert V3\rvert} =$	$\frac{\lvert V2\rvert}{\lvert V3\rvert} =$	$\frac{\lvert H1\rvert}{\lvert H2\rvert} =$	$\frac{\lvert H1\rvert}{\lvert H3\rvert} =$	$\frac{\lvert H2\rvert}{\lvert H3\rvert} =$

What do you notice about the ratios of any 2 vertical line segments of any 2 of these triangles and the ratio of the corresponding horizontal line segments? _____

_____

_____

_____

Student Activity 2

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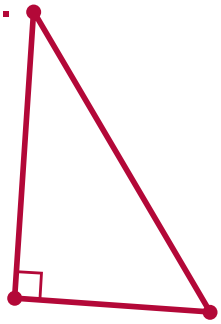
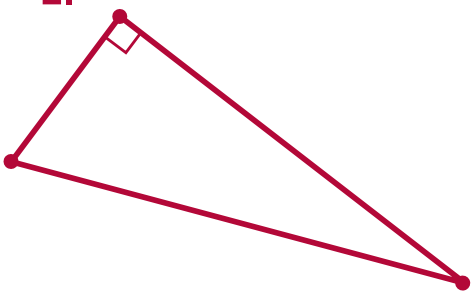
Labelling Sides in Right Angled Triangles

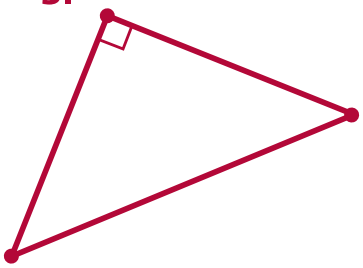
1.



2.



3.



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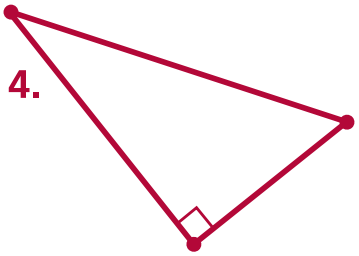
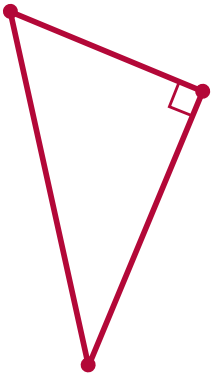
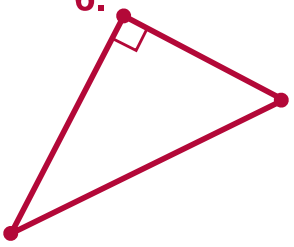


5.



6.



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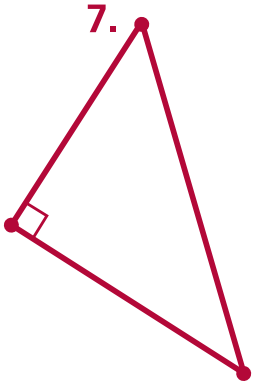

Student Activity 3

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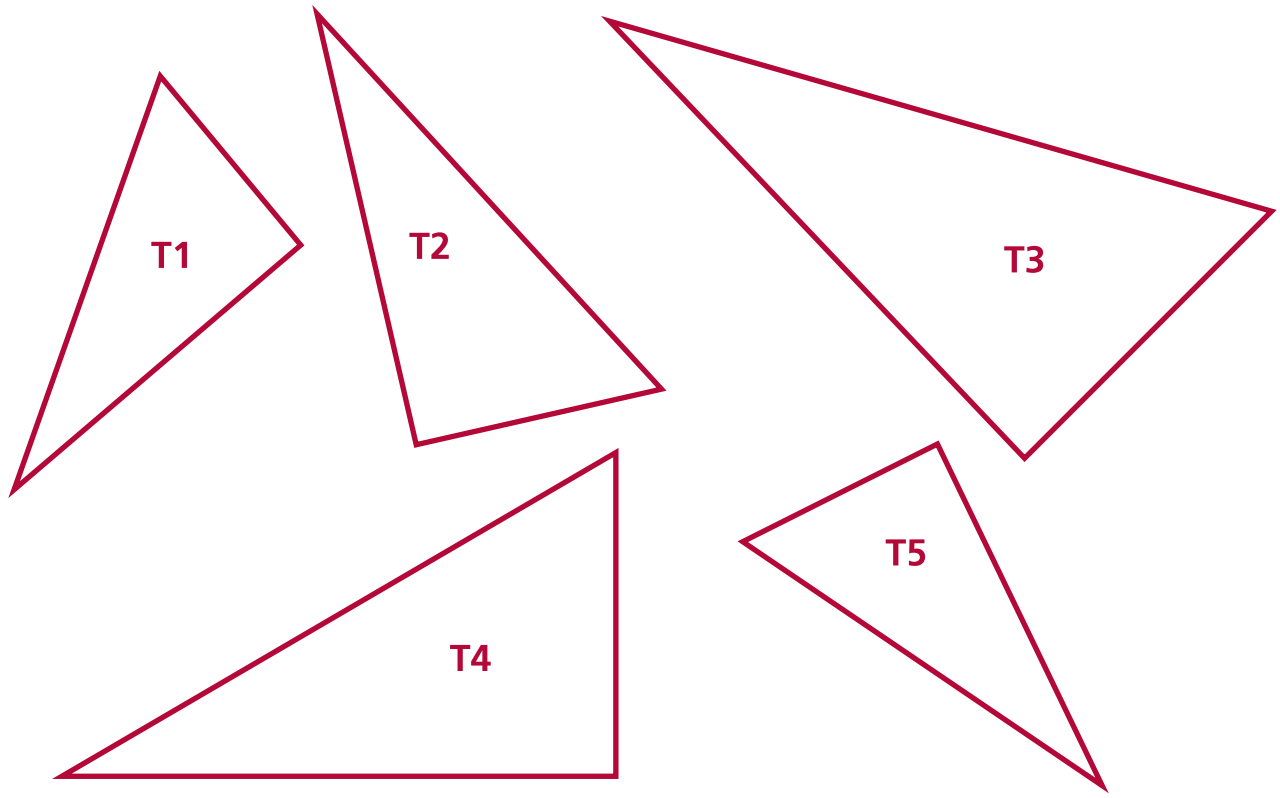
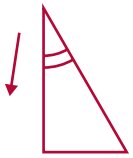
Calculating ratios for similar right angled triangles with angles of 30°

- Measure and label the 90° and the 30° angles in the following triangles. What is the measure of the third angle?
- Label the hypotenuse as "hyp". With respect to the 30° angle, label the other sides as "adj" for adjacent and "opp" for
- Complete the table below.



Marked Angle Size=30° 	|opp|/mm	|hyp|/mm	|adj|/mm	$\frac{opp}{hyp}$ (for angle=30°)		$\frac{adj}{hyp}$ (for angle=30°)		$\frac{opp}{adj}$ (for angle=30°)	
				fraction	decimal	fraction	decimal	fraction	decimal
T1									
T2									
T3									
T4									
T5									
Mean Value (correct to 2 decimal places)									

Student Activity 4

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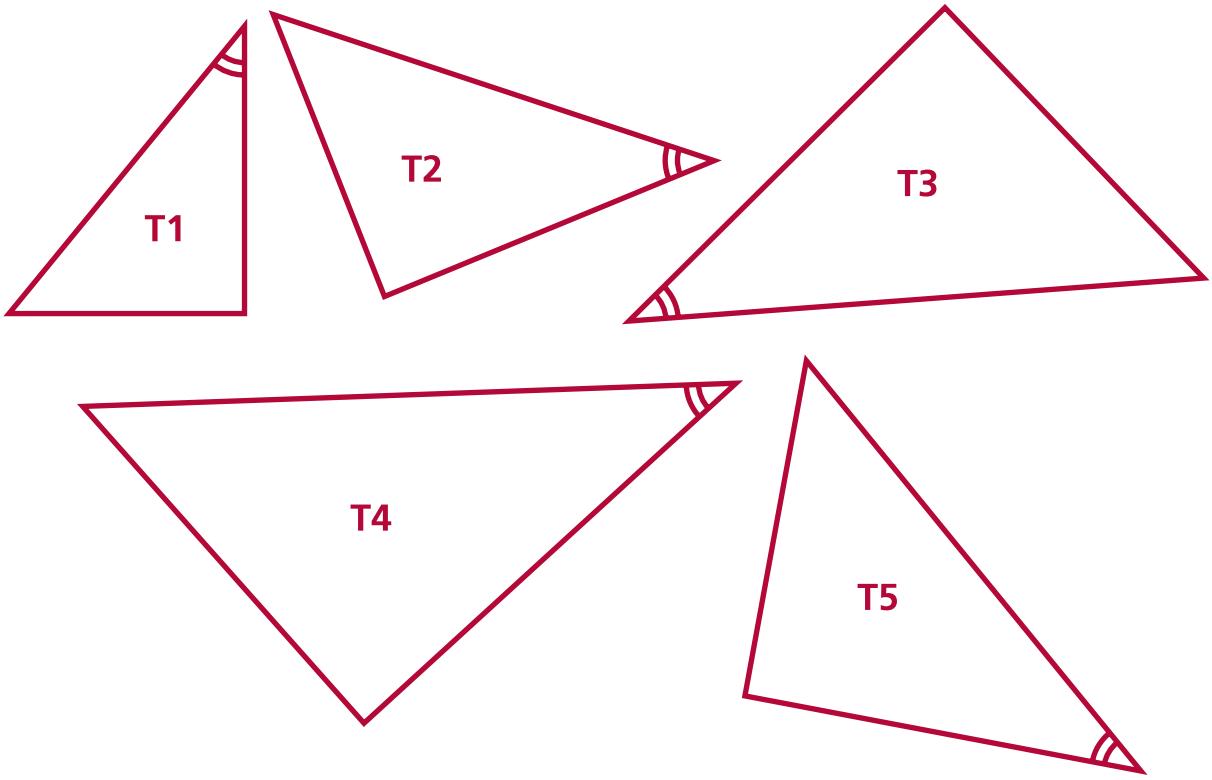
Calculating ratios for similar right angled triangles with angles of 40°

- Measure and label the 90° and the 40° angles in the following triangles. What is the measure of the third angle?
- Label the hypotenuse as "hyp". With respect to the 40° angle, label the other sides as "adj" for adjacent and "opp" for opposite.
- Complete the table below.



Marked Angle Size=40° 	|opp|/mm	|hyp|/mm	|adj|/mm	$\frac{opp}{hyp}$ (for angle=40°)		$\frac{adj}{hyp}$ (for angle=40°)		$\frac{opp}{adj}$ (for angle=40°)	
				fraction	decimal	fraction	decimal	fraction	decimal
T1									
T2									
T3									
T4									
T5									
Mean Value (correct to 2 decimal places)									

Student Activity 5

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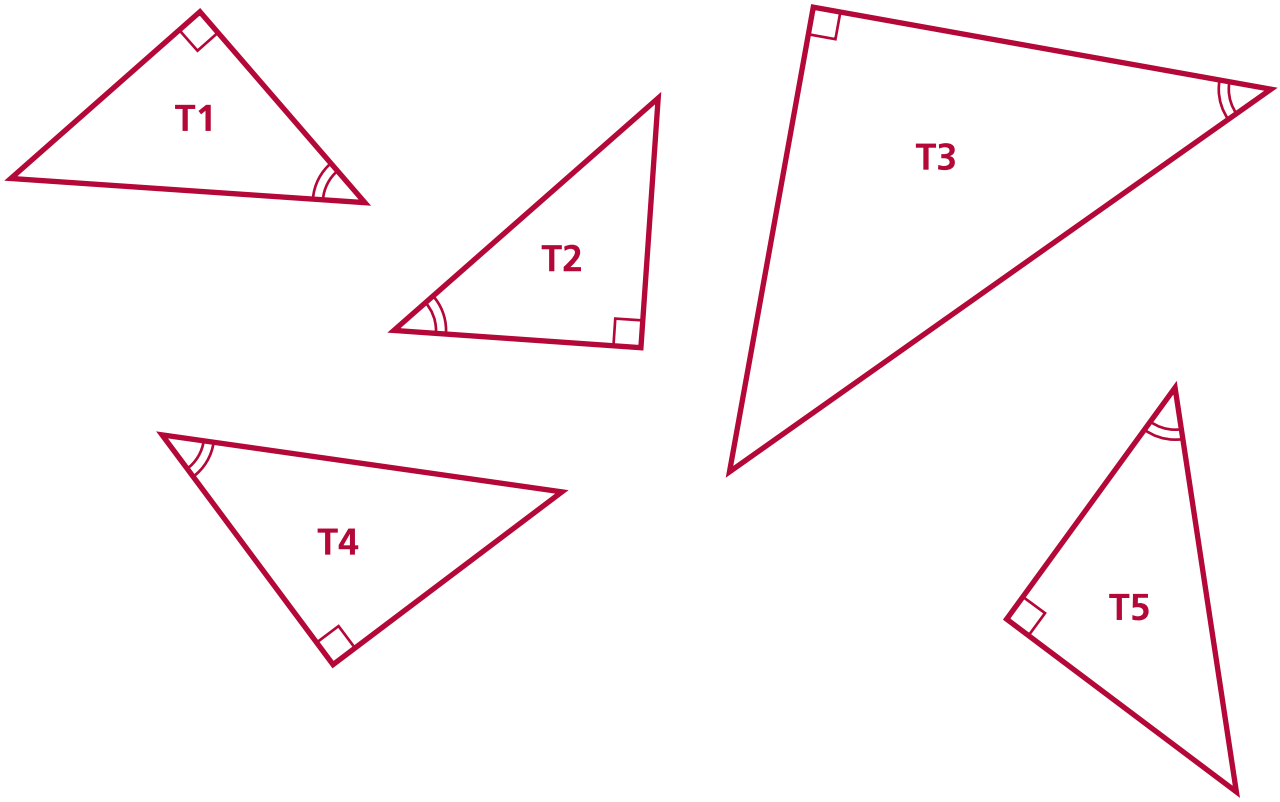
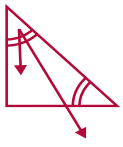
Calculating ratios for similar right angled triangles with angles of 45°

- Measure and label the 90° and the 45° angles in the following triangles. What types of right angled triangle are these triangles?_____
- Label the hypotenuse as "hyp". With respect to the 45° angle, label the other sides as "adj" for adjacent and "opp" for opposite.
- Complete the table below.



Marked Angle Size=45° 	|opp|/mm	|hyp|/mm	|adj|/mm	$\frac{opp}{hyp}$ (for angle=45°)		$\frac{adj}{hyp}$ (for angle=45°)		$\frac{opp}{adj}$ (for angle=45°)	
				fraction	decimal	fraction	decimal	fraction	decimal
T1									
T2									
T3									
T4									
T5									
Mean Value (correct to 2 decimal places)									

Student Activity 6

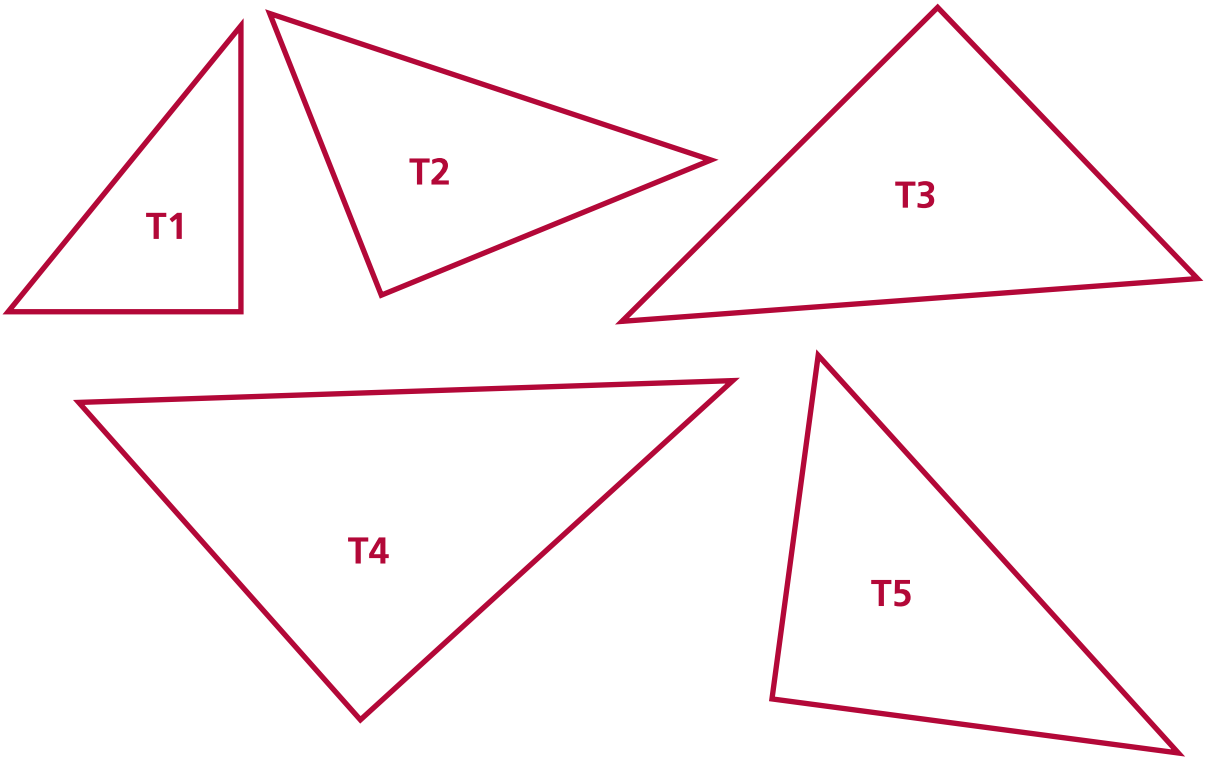
Project Maths Tionscadal Mata Development Team

Calculating ratios for similar right angled triangles with angles of 50°

- Label the 90° and the 50° angles in the following triangles. What is the measure of the third angle?

_____

- Label the hypotenuse as "hyp". With respect to the 50° angle, label the other sides as "adj" for adjacent and "opp" for opposite.
- Complete the table below.



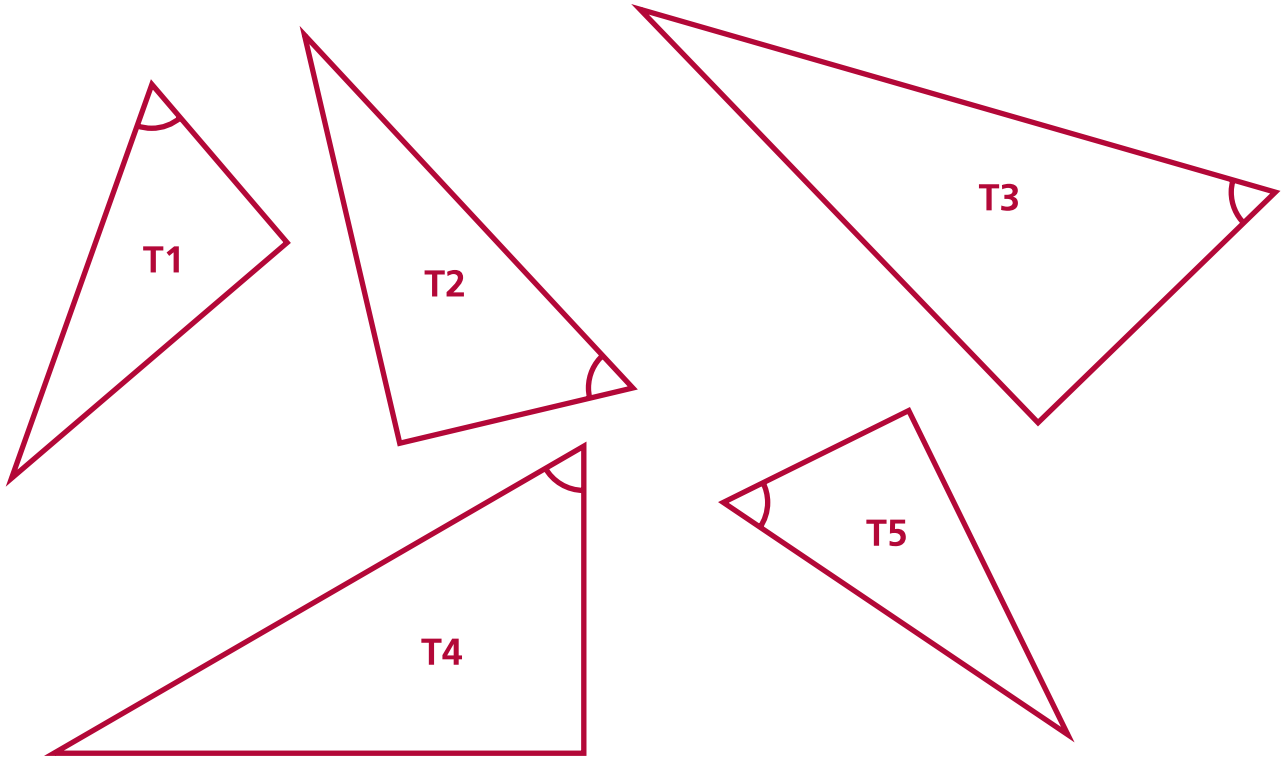
Marked Angle Size=50° 	|opp|/mm	|hyp|/mm	|adj|/mm	$\frac{opp}{hyp}$ (for angle=50°)		$\frac{adj}{hyp}$ (for angle=50°)		$\frac{opp}{adj}$ (for angle=50°)	
				fraction	decimal	fraction	decimal	fraction	decimal
T1									
T2									
T3									
T4									
T5									
Mean Value (correct to 2 decimal places)									

Student Activity 7

Project Maths Development Team

Calculating ratios for similar right angled triangles with angles of 60°

- Measure and label the 90° and the 60° angles in the following triangles. What is the measure of the third angle?_____
- Label the hypotenuse as "hyp". With respect to the 60° angle, label the other sides as "adj" for adjacent and "opp" for opposite.
- Complete the table below.



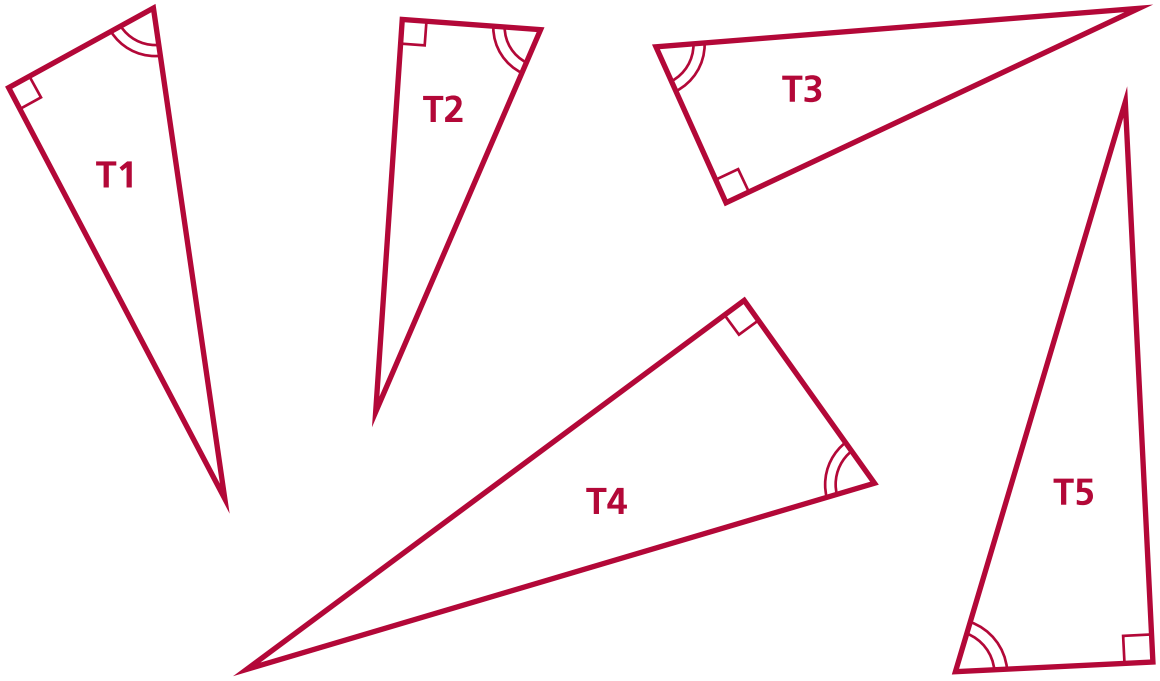
Marked Angle Size=60° 	|opp|/mm	|hyp|/mm	|adj|/mm	$\frac{opp}{hyp}$ (for angle=60°)		$\frac{adj}{hyp}$ (for angle=60°)		$\frac{opp}{adj}$ (for angle=60°)	
				fraction	decimal	fraction	decimal	fraction	decimal
T1									
T2									
T3									
T4									
T5									
Mean Value (correct to 2 decimal places)									

Student Activity 8

Project Maths Development Team

Calculating ratios for similar right angled triangles with angles of 70°

- Measure and label the 90° and the 70° angles in the following triangles. What is the measure of the third angle?_____
- Label the hypotenuse as "hyp". With respect to the 70° angle, label the other sides as "adj" for adjacent and "opp" for opposite.
- Complete the table below.



Marked Angle Size=70° 	|opp|/mm	|hyp|/mm	|adj|/mm	$\frac{opp}{hyp}$ (for angle=70°)		$\frac{adj}{hyp}$ (for angle=70°)		$\frac{opp}{adj}$ (for angle=70°)	
				fraction	decimal	fraction	decimal	fraction	decimal
T1									
T2									
T3									
T4									
T5									
Mean Value (correct to 2 decimal places)									

Student Activity 9

Project Maths Development Team

Master table of class results for ratios of sides in right angled triangles

Angle/°	$\frac{opp}{hyp}$	Check	$\frac{adj}{hyp}$	Check	$\frac{opp}{adj}$	Check
30°						
40°						
45°						
50°						
60°						
70°						

Student Activity 10

Project Maths Tionscadal Mata Development Team

Using the master table of class results answer the following questions

1. What do you notice about sin 30° and cos 60°? _____

_____

2. What do you notice about cos 30° and sin 60°? _____

_____

3. Can you explain what you have noticed using diagrams?

4. How would you describe angles 30° and 60°? _____

_____

5. Can you find similar examples in the master table? _____

_____

6. For what angle in a right angled triangle is the opposite side one half of the hypotenuse? _____

_____

Draw a diagram to illustrate your answer.

7. For what angle in a right angled triangle are the opposite and adjacent sides equal?

_____

8. Calculate $\frac{SinA}{CosA}$ for each angle A. Compare this to the value of Tan A. What do you notice? Can you justify the answer?_____

_____

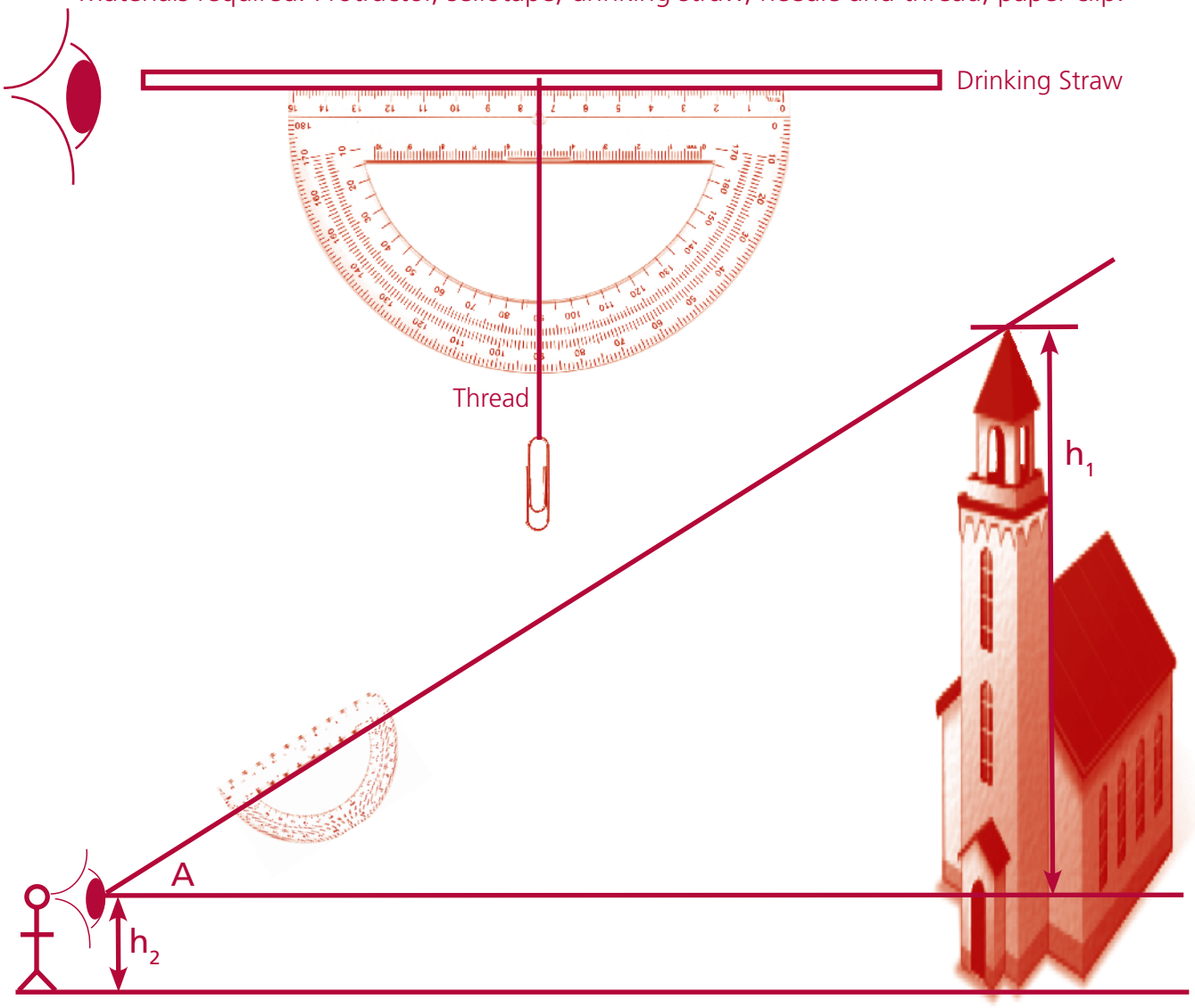
_____

Appendix A

Project Maths Tionscadal Mata Development Team

Making and using a clinometer to find the height of a tall structure

Materials required: Protractor, sellotape, drinking straw, needle and thread, paper clip.



Finding the height of a wall/spire/ flagpole using a clinometer

Work in threes – one holding the clinometer, one reading the angle of elevation, one recording the angle of elevation.

- Measure the height of the observer from eye to ground level.
- Measure the distance from the observer to the base of the building (under the highest point).
- Mark the position of the observer on the ground.
- Hold the clinometer so that the string is vertical.
- Now tilt the clinometer looking through the drinking straw so that the highest point on the top of the wall/flagpole/spire is visible.
- Read the angle of elevation of this highest point to the nearest degree.
- Draw a rough sketch of the situation marking in the distances measured and the angle of elevation.

Appendix A

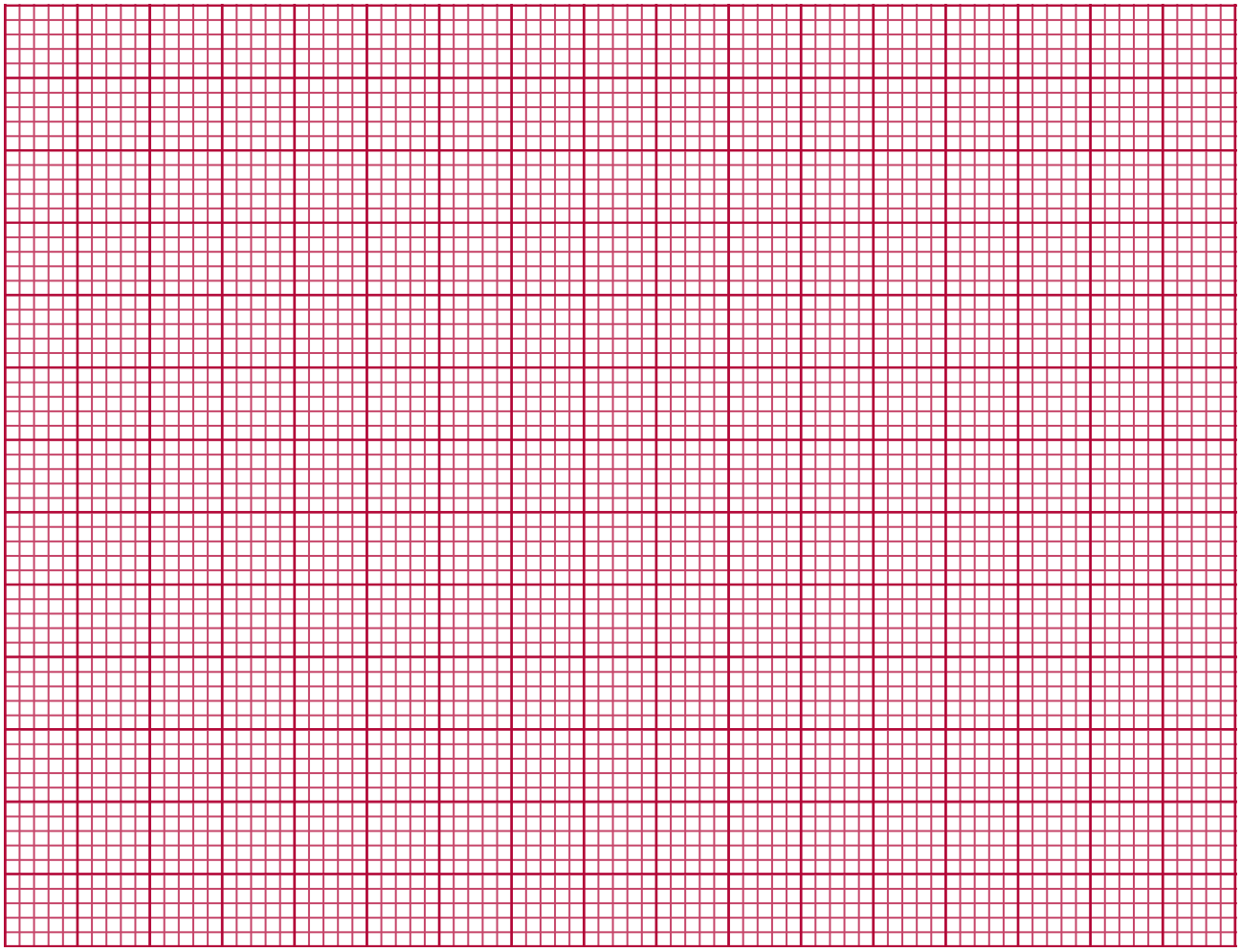
Project Maths Development Team

Rough Sketch

Height from ground to observer's eye	Distance from observer to the foot of the spire	Angle of elevation to the top of the spire

Back in class:

- Calculate the height of a very tall object using a scaled diagram.
- Using graph paper draw a scaled diagram of the above situation. Scale_____



- Measure the height of the spire from the scaled diagram and using the scale factor convert to its actual height.
- Height of the spire above the observer's eye: _____
- Height from ground to the observer's eye: _____
- Height of the spire: _____

Appendix A

Project Maths Development Team

Calculate the height of a very tall object using trigonometry

Redraw the diagram (doesn't have to be to scale) marking the right angle, the hypotenuse, the angle of elevation, the side adjacent to the angle of elevation and the side opposite the angle of elevation. Right angled triangle, sides labelled and measurements included

- What side do you know the length of? _____
- What side do you require the length of? _____
- What trigonometric ratio in a right-angled triangle uses these 2 sides? _____

Using trigonometry, calculate the height of the building.

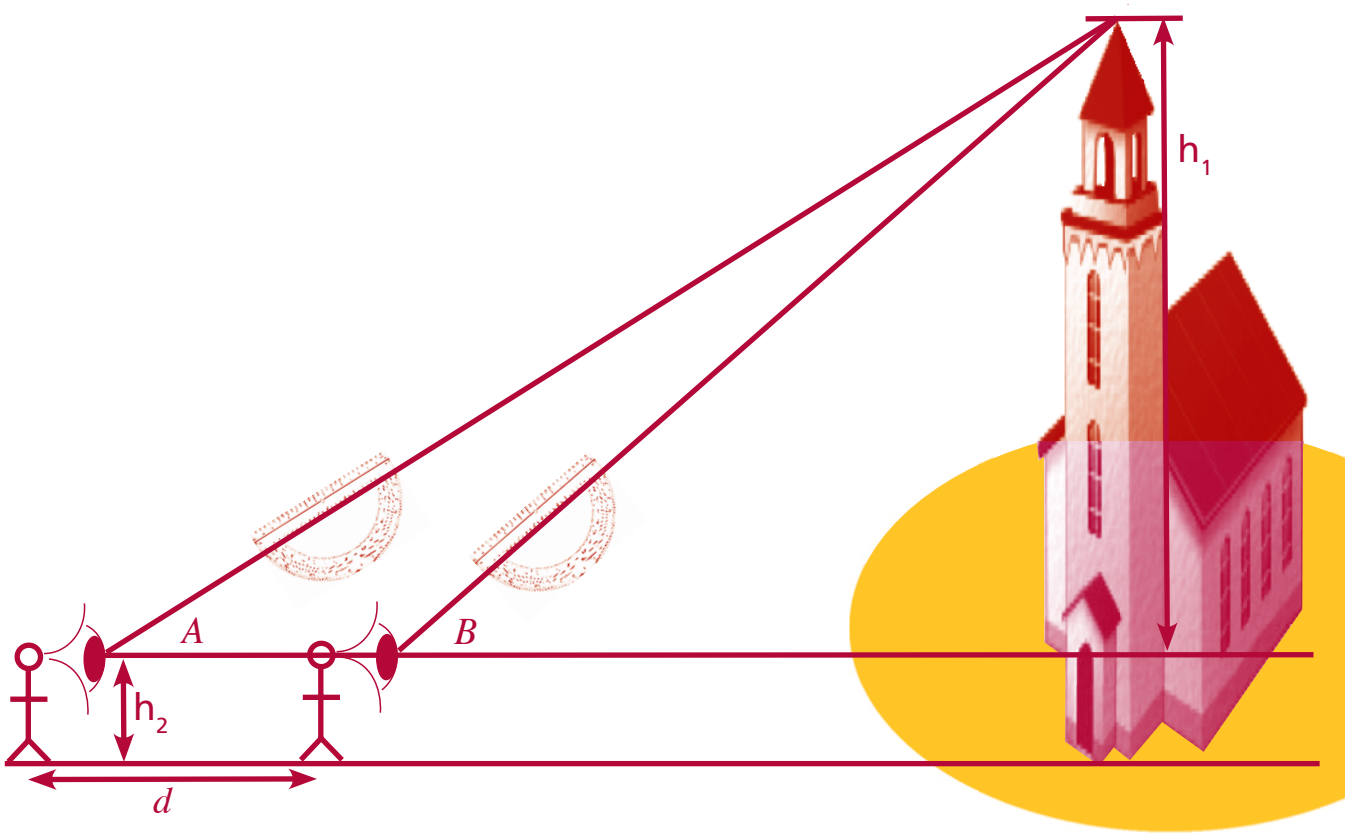
Angle of elevation of the top of the building	Distance to the base of the building d	Tan A = h_1/d	h1	Height of the observer h_2	Height of the building $h_1 + h_2$

Question

As you move towards or away from the building while sighting the top of the spire, the angle of elevation of the top of the spire varies. What angle of elevation would allow the height of the building to be measured by using the distance from the observer to the base of the building added to the observer's height – i.e. no scaled drawing or trigonometry required?

Appendix A

Finding the height of a building with a moat around it



Work in threes – one holding the clinometer, one reading the angles of elevation, one recording the angles of elevation.

- Measure the height of the observer from eye to ground level, h_2, and fill into the table.

h_2	$\lvert <A \rvert$	$\lvert <B \rvert$	$\lvert d \rvert$	h_1	$H = h_1 + h_2$

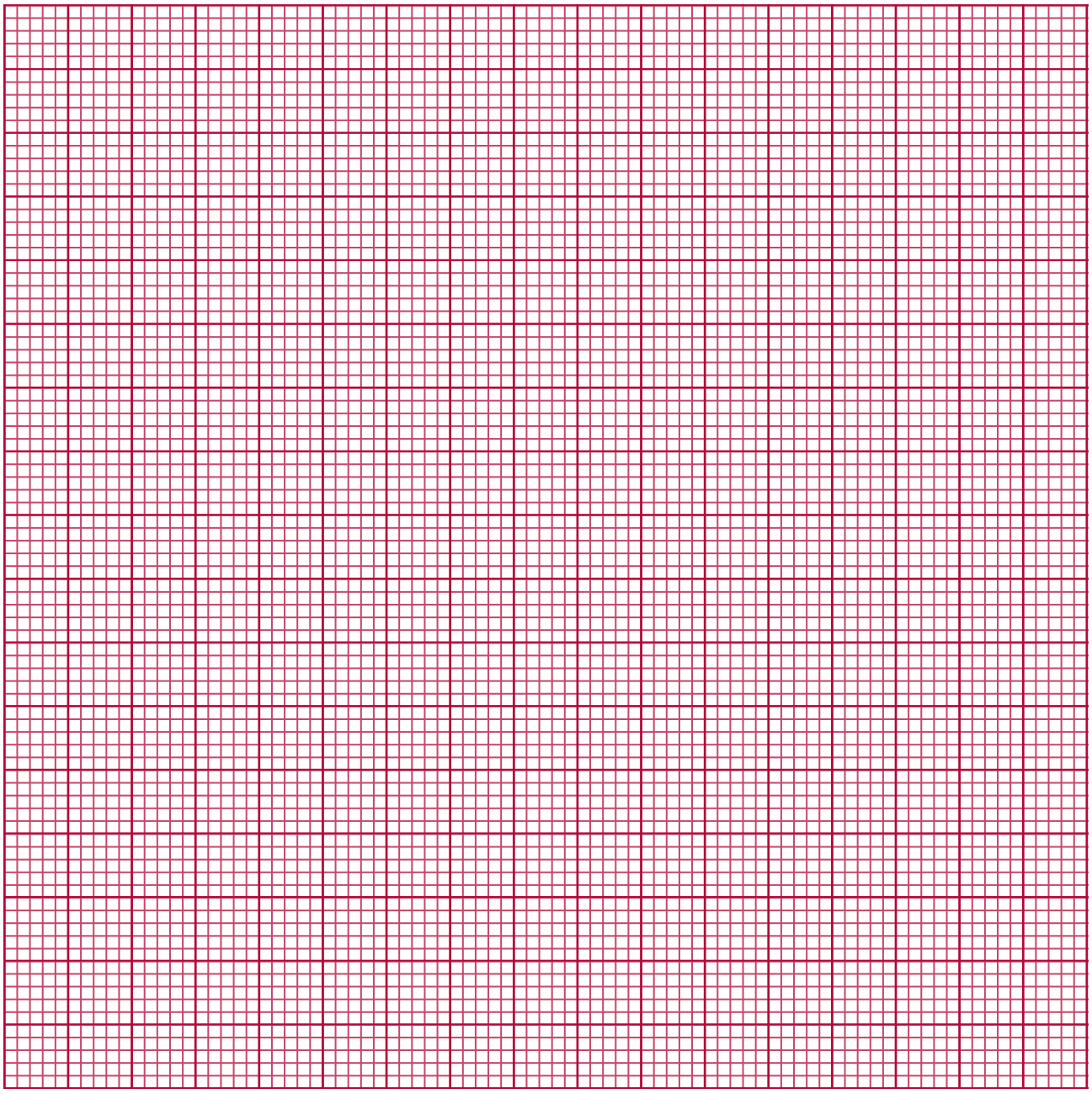
- Mark the position of the observer on the ground.
- Hold the clinometer so that the string is vertical.
- Now tilt the clinometers, looking through the drinking straw so that a point on the top of the wall/ flagpole/spire is visible.
- Read the angle of elevation, <A, to the nearest degree and fill into the table.
- The observer moves closer to the building and again views the top of the building through the drinking straw on the clinometer.
- Read the angle of elevation, <B, to the nearest degree and fill into the table.
- Measure the distance between the two viewing positions, d, of the observer and fill into the table below.
- Draw a rough sketch of the situation marking in the distances measured and the angles of elevations.

Appendix A

Project Maths Development Team

Use a scaled diagram to calculate the height of the building with a moat

Using graph paper draw a scaled diagram of the above situation. Scale _____



- Measure the height of the spire from the scaled diagram and using the scale factor convert to its actual height.
- Height of the spire above the observer's eye: _____
- Height from ground to the observer's eye: _____
- Height of the spire: _____

Appendix A

Project Maths Tionscadal Mata Development Team

Calculating the height of a very tall object surrounded by a moat using trigonometry

Redraw the diagram (which does not have to be to scale) labelling the sides and angles.
Right angled triangle, sides labelled and measurements included

- Fill in all the angle and side measurements known for triangles CBD and CAB.
- Fill in the other two angle measurements in triangle CAB.
- Of the two triangles CBD and CAB, which triangle do you have most information for? _____
- Which side do you require the length of in triangle CBD? _____
- What side is shared by both triangles CAB and CBD? _____
- What rule can be used to calculate this side? _____
- Calculations:

- Label the sides in right angled triangle CBD appropriately as 'hypotenuse', 'adjacent' and 'opposite'.
- Which side do you know the length of? _____
- Which side do you require the length of ? _____
- What trigonometric ratio in a right-angled triangle uses these 2 sides? _____
- Calculations to find the required length:_____

_____

_____

- Complete the table below to find the height of the building H

$\lvert <A \rvert$	$\lvert <B \rvert$	$\lvert d \rvert$	h_2	h_1	$H = h_1 + h_2$

Appendix B